Induced instability for trapped Bose-Fermi mixed condensate due to attractive Boson-Fermion interaction

T. Miyakawa, T. Suzuki and H. Yabu

Physics Department, Tokyo Metropolitan University
1-1 Minami-Ohsawa, Hachioji, 192-0397, Tokyo, Japan
Tel +81-426-77-1111 (3381), Fax +81-426-77-2483
E-mail: tmiya@comp.metro-u.ac.jp

Discoveries of the Bose-Einstein condensates (BEC) for Alkali atoms and the Fermi degeneracy for trapped ^{40}K atoms encourage the study for boson-fermion mixed condensates of trapped atoms. In this study, we focus on the instabilities and collapses of the boson-fermion mixed condensates due to the attractive boson-fermion interaction[1]. In BEC, such instabilitiy has been observed in the trapped meta-stable ^{7}Li.

We consider the polarized boson-fermion mixed condensate of potassium isotopes in the spherical harmonic potential at $T = 0$. Using the Thomas-Fermi approximation for fermion degree of freedom (appropriate for large fermion number), the total energy of the system becomes a functional of the boson order-parameter $\Phi(\vec{r})$ and the fermion density distribution $n_f(\vec{r})$:

$$E[\Phi(\vec{r}), n_f(\vec{r})] = \int d^3r \left[\frac{\hbar^2}{2m}|\nabla\Phi(\vec{r})|^2 + \frac{1}{2}m\omega^2\vec{r}^2|\Phi(\vec{r})|^2 + \frac{g}{2}|\Phi(\vec{r})|^4\right]$$
$$+ \int d^3r \left[\frac{3}{5}\frac{\hbar^2}{2m}(6\pi^2)^{2/3}n_f^{5/3}(\vec{r}) + \frac{1}{2}m\omega^2\vec{r}^2 n_f(\vec{r}) + h|\Phi(\vec{r})|^2 n_f(\vec{r})\right], \quad (1)$$

where ω and m are the angular frequency for boson/fermion trapping potential and a boson/fermion mass. Boson-boson (boson-fermion) interaction strength $g(f)$ is represented by its s-wave scattering lengths, while the fermion-fermion interaction has been neglected for the polarized system.

Evaluating the total energy with the variational method, we take the Gaussian ansatz for boson order-parameter:

$$\Phi(r; R) = \sqrt{\left(\frac{3}{2\pi}\right)^{3/2}\frac{N_b}{R^3}} \exp\left(\frac{-3r^2}{4R^2}\right). \quad (2)$$

The variational parameter R in (2) corresponds to the root-mean-square (rms) radius of the boson density distribution. In the Thomas-Fermi approximation with (2), the fermion density distribution $n_f(\vec{r})$ becomes

$$n_f(r; R) = \frac{1}{6\pi^2}\left(\frac{\hbar^2}{2m}\right)^{3/2} \left\{\mu_f(R) - r^2 - h|\Phi(r; R)|^2\right\}^{3/2} \theta(R_f - r), \quad (3)$$

where R_f is the root of the quantity inside curly braket. Using eqs. (2) and (3), the total energy $E(R) = E[\Phi(r; R), n_f(r; R)]$ becomes a function of a single parameter R.

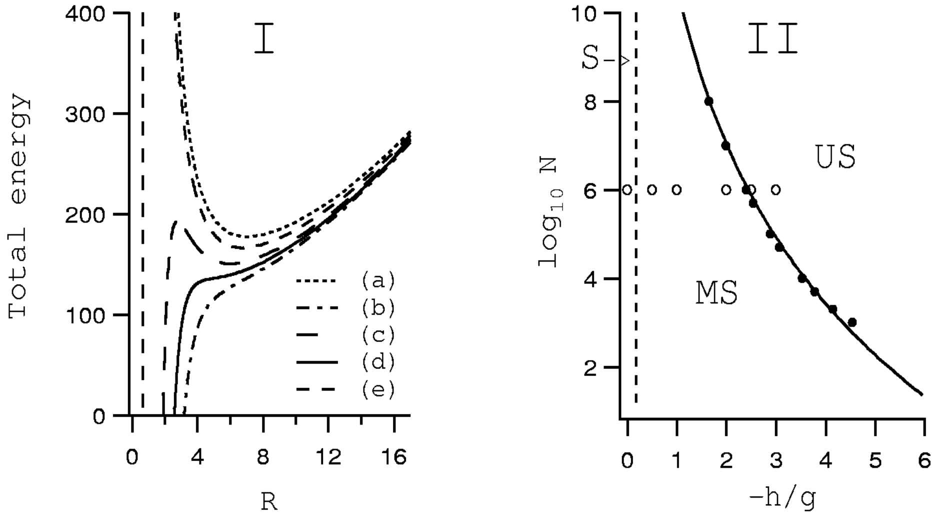

Figure 1: I: Total energy variation $E(R)/(N\hbar\omega)$. II: Stability phase diagram for N and $-h/g$. Boson-boson scattering strength is 15.13 [nm] and $\omega = 450$ [Hz] ($N = N_b = N_f$).

In Fig. 1-I, numerical results for $E(R)$ are plotted for $\alpha \equiv -h/g = 0.1, 1.0, 2.0, 2.5, 3$ (lines a-e) with $N = 10^6$, where three kinds of patterns can be read off: stable (a), meta-stable (b,c) and unstable states (d,e). For weak boson-fermion interaction, the system is stable against the variation of R, and has an absolute minimum as an equilibrium state. For the intermediate strength, the system becomes meta-stable with one local minimum (lines b,c), and, in strong attractive interactions, the minimum disappears and the system become unstable (lines d,e).

As shown in Fig. 1-I, the stability of mixed condensate can be judged from the small R behavior of $E(R)$: positive divergence in small R implies astable state. To obtain the stability condition, we consider analytically the asymptotic expansion of $E(R)$ in $R \ll 1$. The leading order of $E(R)$ in small R gives the stability critical condition : $-h/g|_c = 2^{-5/2}(N_b/N_f)$(Fig. 1-II dashed line).

On the other hand, the critical condition for unstable regions is obtained with the Thomas-Fermi approximations for both fermion and boson distribution. Especially, if the boson and fermion chemical potensials satisfy the condition : $\alpha\mu_b \ll \mu_f \sim \mu_f(h = 0)$, the instability critical condition becomes $h^2/g|_c = (4\pi^2/6^{2/3})N_f^{-1/6}$(Fig. 1-II solid line). In the variational method, we evaluated the critical α between the meta-stable and unstable states for several N; the results are plotted in Fig. 1-II (filled circles).

Finally, in the collapses of metastable and unstable states, both boson and fermion distributions become localized and compressed. Those collapses can also be determined from instability of the in-phase monopole excitation evaluated in the sum-rule method [2].

[1] T.Miyakawa, T.Suzuki and H.Yabu, cond-mat/0002048

[2] T.Miyakawa, T.Suzuki and H.Yabu, cond-mat/0002145